Supporting Collaborative Learning of Probabilities with a Tangible User Interface: Design and Preliminary Results

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Abstract: In this paper we introduce *Combinatorix*, a learning environment for supporting collaborative problem-solving in probability. The tabletop system has two working spaces where users perform different actions: on the horizontal surface, users combine tangible objects in different orders and watch the effects of various constraints on the problem space. A vertical display shows an abstract representation, such as a probability tree, to reflect how users' actions influenced the total number of combinations. We describe our iterative participatory design process with college students taking a probability class, as well as their informal feedback with the final system. We discuss the benefits and the challenges of our approach and conclude with an analysis of how tabletops and tangible user interfaces can affect education.

INTRODUCTION

Tversky and Kahneman [11] demonstrated that everyone, even professional statisticians, suffer from systematic biases in their intuitive judgments of probability. Yet understanding probability is essential for dealing with everyday life. Politicians often make policy without understanding the implications of the data available to them. Patients have difficulty interpreting medical test results or evaluating the costs and benefits of a vaccine. Batano and Sanchez [2] identified a variety of student mistakes when solving probability problems, including the gambler's fallacy, data representativeness, and equiprobability bias. Even graduate students who plan to teach mathematics maintain strong misconceptions [7]. More importantly, a lot of classic mathematical problems can be seen and solved as probabilistic processes [12].

Tabletop systems have been used successfully to help users manipulate virtual concepts as if they were physical objects [3]. Our challenge is to use tangible interaction to support learning of complex abstract concepts that involve a combinatorial explosion, in which even simple problems result in hundreds of possibilities. Combinatorics is a branch of probability that deals with the enumeration, combination, and permutation of sets of elements and their mathematical relationships. We believe that combined with existing frameworks on teaching probabilistic thinking [1], those technologies have the potential to impact the way students learn combinatorics, probability and statistics.

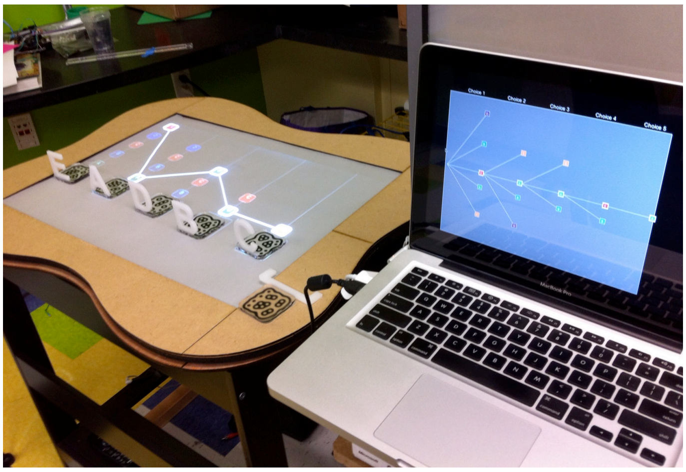

Figure 1. Combinatorix: Tangible elements control a tabletop display (left) with corresponding probability tree (right)

This paper describes the design and development of Combinatorix (Fig.1), a tangible tabletop interface in which users manipulate physical objects to obtain deeper insights into complex mathematical relationships. Our goal is not to transform virtual into physical objects but rather *to use physical objects to explore fundamentally abstract concepts in combinatorics.*

Theoretical Framework

Combinatorix is not designed to teach probability per se, but rather to provide a learning environment that encourages small groups of students to explore and discuss combinatorics problems. They should be able to express ideas and hypotheses, struggling with concepts in a productive way. Ideally, they will build their own theories, appreciate the challenges of defining an elegant formula and understand what their personal strengths and weaknesses are.

We support Fast's [4] constructivist approach, which emphasizes that "overcoming misconceptions through supportive frameworks such as a series of anchoring situations". Our approach builds upon the "Preparing for Future Learning" (PFL) framework [9] in which students begin by analyzing contrasting examples of a concept to isolate important "deep features" of a combinatorics problem, in contrast to the "surface features" or superficial characteristics of a model [6]. Rather than limiting the number of cases, students should be able to express a variety of cases, each with their own visual representations.

The learning environment should provide students with tools for reasoning about probabilities, including visualizations that support their reflections. Students should be able to associate common features of a problem with accepted mathematical representations, e.g., a probability tree. This implies that students need two interactive spaces: one for manipulating concrete, physical objects to explore the problem space and one for displaying the corresponding abstract representation of the problem space. The specific learning goals are for students to:

1. learn the concepts of sample and event spaces, with probability defined as a ratio of the two;
2. compute sample and event spaces using factorials, permutation and combinations with various constraints; and
3. identify the deep structure of a problem as a probability tree and transfer this understanding to new situations.

Methods: Participatory Design Study

The original motivation for this project stemmed from observations of students in a University-level course in combinatorics. Faced with only paper and pencil, many had difficulty developing intuitions about probabilities [5] and suffered from the 'stereotype threat' [10] that they are poor in math. We hoped that letting students manipulate concrete objects while simultaneously observing the corresponding changes in deep structure, e.g. a probability tree, would reinforce their intuitions about the underlying mathematical principles. Our goal was to create an engaging and playful environment that avoids excessive mathematical notations and encourages discussion.

We began by conducting ten one-hour semi-structured interviews with students currently enrolled in a probability class. We found that less proficient students:

- crave concrete examples and visualizations,
- attempt but often fail to create their own representations, due not only to their lack of domain expertise but also to the limitations of pen and paper: one cannot draw a probability tree with 100 leaves,
- jump too quickly to abstract representations, e.g., formulas and mathematical notations, a major barrier to conceptual understanding,
- experience anxiety and cognitive load when faced with mathematical notations, and
- do not know where to start, often asking the teaching assistant to effectively solve the problem for them.

We next created a mockup with cardboard letters representing the building blocks of combinatorial problems. Participants could address questions such as: How many possible combinations of A, B and C are there? We also provided cardboard constraints to address questions such as: How many combinations obtain if A and B must be next to each other? Participants formed questions by combining physical letters and we created a corresponding visual representation (Fig. 2) with paper or on a whiteboard. One student suggested an innovative visualization, a kind of fractal representation that we tried with other students (Fig. 2, bottom). Based on these explorations, we designed Combinatorix, a custom-made tabletop with tangible objects that students manipulate to express and explore combinatorial problems.

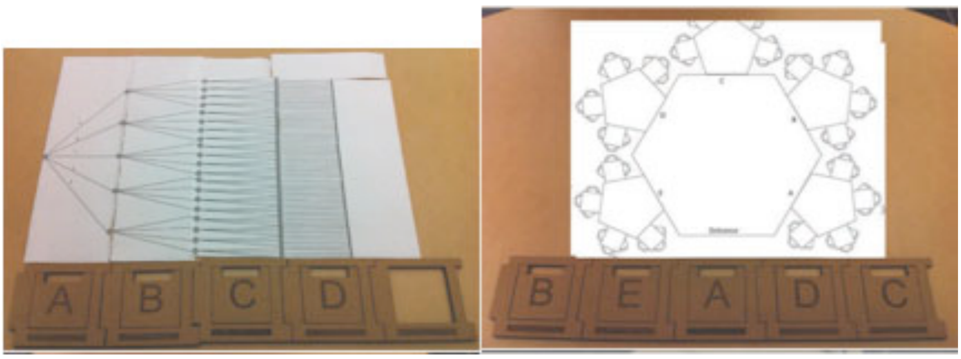

Figure 2. Participatory design: Cardboard mockup with paper-based tree (top) and graph representations (bottom)

Combinatorix

Hardware

Combinatorix (Fig. 4) supports several input techniques: a camera detects the location of fiducial markers and a wiimote provides the position of multiple infra-red pens. A projector displays additional information around the tangible objects. The interactive surface is 60 x 45 cm. and can accommodate up to four students at the same time.

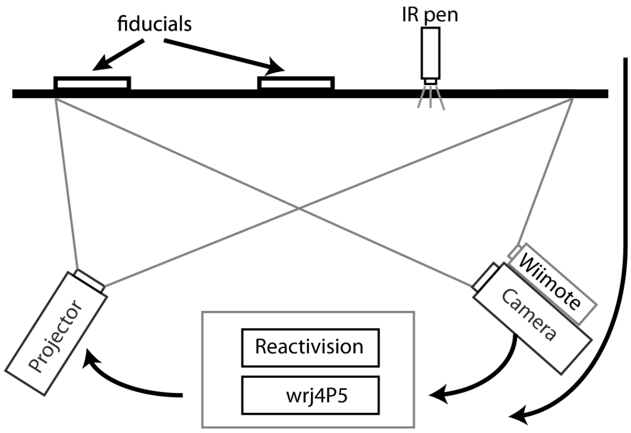

Figure 3. Combinatorix setup: The webcam detects location of fiducial markers; the wiimote detects position of infra-red pens

Software

The underlying application is written in Java and uses the Reactivision engine to detect fiducial makers [8]. Additional libraries, e.g., wrj4P50, communicate with the wiimote. The system is modular and can easily accommodate the creation of additional operators for constraining the sample space.

The current version displays two kinds of information: first, the tabletop interface shows a specific number of placeholders for objects. Letters can be placed on those spots to form a new combination. At the same time, the remaining number of letters for each step is displayed on top of each placeholder. A second screen displays a probability tree reflecting the current state of the problem. Letters can easily be replaced by other elements, including virtual, laser-cut and 3D-printed physical objects. Combinatorix supports up to 10 tangible objects and 20 virtual ones.

Interaction Techniques

Students can interact in two ways: 1) Use tangible letters to form combinations or to add constraints, e.g., fixing the position of a particular element. For example, Fig. 4 (top) shows the number of combinations when A and B are attached to each other. 2) Use a pen to annotate the probability tree. For example, Fig 4 (bottom) shows how to "prune" certain sections, which is equivalent to dividing a factorial number with the combinations that don't satisfy the constraint.

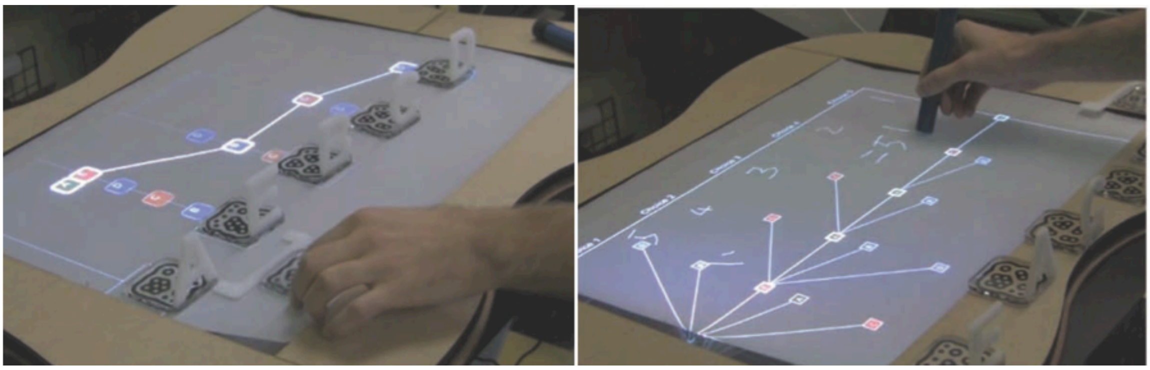

Figure 4. Combinatorix lets users switch between two modes: constructing a combination using physical objects (left) and annotating a probability tree (right).

Preliminary Results

Five participants tested Combinatorix, including two high-school students and three university students. We asked them to use the table to solve five problems of increasing difficulty: "The letters A, B, C, D, E form how many different linear arrangements?" 1) in total, 2) for which A and B are next to each other, 3) where E is not last in line, 4) for which A is before B, and 5) where A and B are next to each other and C is not first in line?".

General reception

Users were enthusiastic about using the system to solve the problems and were generally able to come up with the right solution after a few minutes. Problem four was the most difficult since the system does not provide any relevant hints. Instead, students tried a brute force solution, exhaustively counting the number of possible cases. The university students eventually realized that the problem was about symmetry: there is an equal number of combinations in which A is before B and B is before A. The solution is thus to divide the total number of combinations by two, e.g. 5! / 2. High school students required more support, in the form of prompts from the experimenter, to find this solution. Such prompts could easily be integrated into the system as automatic feedback; for instance, if a student spends too much time on a particular problem, Combinatorix could display a small hint to unblock the situation.

Participants found the current prototype very useful for certain types of problems: Ann[1] noted that "All the functionalities you could add should not do the thinking for the student; if I use this piece, it's telling what the solution is... well not really. It's more like a hint". Interviewer: "So do you think it's too much help?"; Ann: "I think it's a good level of help, because it conveys the notion that in this situation there are only four combinations that can be here [...]".

However, Combinatorix clearly does not support all types of combinatorics problems. Henry said that "this is a really elegant way to show the concept of factorials; but for some problems I feel like I need to already know that concept to figure it out to get the solution". He also noted "it would be excessive to build a new model for each problem". This is the main challenge for our approach: some classes of problems can be supported easily, but others might require a totally different interface.

Although Combinatorix currently supports high-school level problems, future versions will address college-level problems including conditional probabilities (Bayes' theorem), independence of events, statistical indices (expected value, variance, standard deviation), discrete distributions (binomial, multinomial, geometric, hypergeometric, negative binomial), continuous distributions (uniform, normal, exponential, beta), law of large numbers and central limit theorem. We plan to support specific problems, such as the ones described, rather than creating a fully open-ended system, providing additional scaffolding to extend basic functions.

Our next step is to deploy the Combinatorix prototype, with additional problem types, during the office hours of a university-level probability class. Students will be able to interact with the system with or without the help of teaching assistants.

Conclusion

Due to the complexity of the domain i.e. combinatorics, and more generally probability, we do not envision Combinatorix as a stand-alone teaching tool. Rather, we consider it as a platform for students to reflect on problems, offload the cognitive burden of picturing all possible options and as a tool to provide small hints when students are stuck on a problem. Initial user testing revealed that students thought of it as a useful tool, but also mentioned important challenges that need to be addressed. For space considerations, this is a sample of the data and it tells us that Combinatorix has the potential to engage students in probabilistic thinking. For the final conference paper, we will include data from additional subjects and discuss how physical actions impact students' understanding of combinatorics.

For future versions of Combinatorix, we intend to evaluate its potential as a collaborative tool in a formal learning setting. Moreover, we will organize additional participatory design sessions to revise our prototype and gain additional insights from users. Finally, we plan on more formally assessing its effectiveness by conducting a series of controlled experiments.

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[1] All participant names have been anonymized.

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